The MOTA Framework: A Whitepaper on the Dynamic Optimization of Bankroll Growth in Tournament Poker

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ABSTRACT

This paper introduces the Mean Optimised Tournament Alpha (MOTA) Framework, a comprehensive methodology that provides a formal structure for decision-making under the profound uncertainty inherent in professional poker. The framework's purpose is to generate "Tournament Alpha"—an Expected Growth Rate (EGR) superior to what is achievable through conventional strategies. We first establish that any decision to participate in a tournament is a fractional betting strategy, for which the Kelly Criterion is competitively optimal in an asymptotic sense. The MOTA Framework's superiority is then formally demonstrated. It does not propose an alternative to the Kelly Criterion; rather, it provides a mechanism for systematically engineering an investment asset with a superior risk/return profile, to which an optimal betting fraction can then be applied. By optimizing over this expanded set of engineered investment opportunities, the framework achieves a maximal EGR that is provably superior to or equal to the optimal EGR attainable on the raw investment alone.

Keywords: Kelly Criterion, Portfolio Optimization, Decision-Making under Uncertainty, Bankroll Management, Asset Allocation, Fat-Tailed Distributions

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1. Introduction and Literature Review

The central problem for a professional poker player is the optimal allocation of a finite bankroll under uncertainty. The established theoretical solution is the maximization of the logarithm of wealth (Kelly Jr., 1956), which maximizes the asymptotic compound growth of capital (Cover & Thomas, 2006; Thorp, 2006). However, the direct application of this "Pure Kelly" strategy, a myopic policy, can lead to excessive volatility (MacLean, Thorp, & Ziemba, 2010), a problem exacerbated by the high-variance, fat-tailed return distributions of poker tournaments (Lantz, 2015). This paper provides a structural enhancement to this process by separating the problem of asset engineering from the problem of optimal bet sizing.

2. The MOTA Framework: A Three-Stage Process

2.1 Stage I: The Prerequisite of Distributional Estimation

The foundation of the **MOTA framework** is a player-specific probability distribution , often encapsulated by their estimated Return on Investment (ROI) for a given event. The framework is a deterministic engine that processes these inputs; its output is therefore entirely dependent on their quality. This relationship is subject to the principle of "Garbage In, Garbage Out" (GIGO). The methods for generating accurate, well-calibrated distributional inputs are complex and constitute a significant source of a professional's competitive advantage. As such, the specific techniques for predicting performance fall outside the scope of this paper.

2.2 Stage II: Investment Engineering and Formal Optimization

This stage formalizes the act of selling fractional equity ("action") as a form of financial engineering. The framework's dominance is established by the principle that an optimization performed over an expanded set of possibilities (i.e., all potential sale percentages $s \in [0, 1]$) must yield a result at least as good as, and potentially superior to, an optimization over any subset of those possibilities.

The core of this engineering process is to solve for the optimal sale percentage, s^*, that maximizes the expected logarithm of wealth. For a simplified single-payout tournament structure, the solution can be derived analytically. Given the variables B (Bankroll), C (Buy-in), P (Prize), p (Win Probability), and m (Markup), the optimal sale percentage s^* is given by the formula:

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s* = [ (B - C) * (p*P - C*m) - (1-p) * C*m*P ] / [ C*m * (C*m - P) ]
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This equation provides the precise fraction of action to sell to achieve the highest possible Expected Growth Rate. While presented for a single-payout case, the underlying principle extends directly to complex, multi-payout structures, where the solution is found using numerical optimization.

2.3 Stage III: Opportunity Selection via Time-Normalized Growth

The final stage introduces a universal metric for comparing the now-optimized opportunities: bankroll growth per average hour of tournament duration, calculated as `EGR_mota / T_avg`.

3. Application: The Professional's Dilemma - A Worked Example

To compare different investment strategies (e.g., varying sale percentages), a metric is needed that captures not just potential profit, but also risk. Simple Expected Value (EV) is insufficient, as it ignores the impact of variance on long-term bankroll growth. The theoretically sound objective is to maximize the compound growth rate, which is equivalent to maximizing the Expected Logarithm of Wealth ($E[\log(W)]$).

However, `E[log(W)]` is an abstract number. To make our comparison intuitive, we use the **Certainty-Equivalent Bankroll (`W_ce`)**. Calculated as `exp(E[log(W)])`, the `W_ce` translates the abstract growth potential of a strategy into a tangible dollar figure. It answers the question: *"What is the risk-free amount of money that provides the same utility as this gamble?"* By choosing the strategy with the highest `W_ce`, we are, by definition, selecting the path with the optimal balance of risk and reward for maximizing long-term wealth. This makes it the ideal metric for our analysis. A glossary of terms is available in **Appendix A**.

Setup:

- **Player Bankroll (B):** $15,000
- **Player True ROI:** 30%
- **Tournament:** $1,000 buy-in, 5-person winner-take-all ($5,000 prize)
- **Staking Market:** A markup of 1.2 is available (implying a market-perceived 20% ROI).

Analysis: We compare the `W_ce` for three distinct strategies. The detailed numerical calculation is shown in **Appendix B**.

Strategy (Amount Sold, s)	W_ce (Certainty Equivalent)	Analysis
Play 100% (s=0)	$15,155	The Benchmark. This strategy maximizes expected value but its high variance suppresses the long-term compound growth rate.
Sell 100% (s=1)	$15,200	The Pure Arbitrageur's Play. A risk-free, positive-EV option that is superior to the high-risk benchmark in terms of geometric growth.
MOTA Optimum (s≈66.7%)	**$15,216**	**The True Optimum.** This partial sale creates a "synthetic asset" with an optimally shaped risk/return profile. It achieves the highest possible geometric growth rate for the given opportunity set.

4. Discussion: The True Nature of the MOTA Framework

The worked example reveals the precise nature of the MOTA Framework. It is not an alternative to Kelly betting, nor is it a risk-mitigation strategy in itself. The framework's singular contribution is that it provides a mechanism to **construct a superior asset**.

It is crucial to distinguish the MOTA framework's objective from that of risk-mitigation overlays like "fractional Kelly." The framework's sole purpose is to identify the asset with the highest *possible* Expected Growth Rate (EGR_max). MOTA optimizes the *asset*; the player's risk tolerance determines the betting *aggression*.

Furthermore, the framework's output is dynamic. As a player's bankroll grows, a fixed buy-in represents a smaller fractional risk, reducing the incentive to sell a stake. Similarly, as the delta between the player's true ROI and the market-implied ROI widens, the opportunity cost of selling action increases. In these scenarios, the framework's recommendation logically converges towards the "Pure Kelly" strategy of playing 100% of one's own action ($s \to 0$), demonstrating its robustness across a full spectrum of conditions.

5. Conclusion

The MOTA Framework provides a methodology that is provably superior to conventional, discrete strategies. It formally validates the emergent behavior of elite professionals by situating it within the meta-game of portfolio optimization. The framework does not seek to replace the time-tested principles of optimal growth theory, but to enhance their application. By separating the strategic engineering of an investment from the tactical execution of a bet, it provides a complete, rational pathway from a player's defensible beliefs to the most efficient allocation of their finite resources.

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6. References

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Appendix A: Glossary of Technical Terms

- **Alpha (Tournament Alpha):** The Expected Growth Rate (EGR) generated by the MOTA Framework in excess of the EGR of the benchmark strategy.
- **Certainty-Equivalent Bankroll (W_ce):** The guaranteed cash amount an individual would consider of equal value to a risky proposition. For the logarithmic utility model, it is calculated as `exp(E[ln(W)])`, where `E[ln(W)]` is the Expected Log of Wealth.
- **Expected Growth Rate (EGR):** The primary metric for optimization. Maximizing `E[ln(W)]` is mathematically equivalent to maximizing the long-term compound growth rate of a bankroll.
- **Kelly Criterion:** A formula used to determine the optimal size of a series of bets to maximize the long-term growth rate of a bankroll.
- **Logarithmic Utility:** A model of wealth that reflects diminishing marginal utility, providing the mathematical foundation for risk aversion in growth models.
- **Markup:** The premium at which a player sells tournament action. A markup of 1.2 on a $1,000 buy-in means a backer pays $1,200 for 100% of the player's potential winnings.
- **Return on Investment (ROI):** A measure of profitability, calculated as `(Net Profit / Investment Cost).`
- **Synthetic Asset / Investment Engineering:** The core process of the MOTA Framework, where a player actively alters the financial structure of an investment by combining the raw tournament entry with a financial transaction (selling equity).

Appendix B: Numerical Calculation of the MOTA Optimum

This appendix details the derivation of the optimal sale percentage (`s`) from the worked example, applying the principles from Stage II.

1. Define Variables and Initial State

- Initial Bankroll, `B`: $15,000
- Buy-in, `C`: $1,000
- Prize Pool, `P`: $5,000
- Player's Win Probability, `p_win`: 0.26 (from 30% ROI)
- Player's Lose Probability, `p_lose`: 0.74
- Market Markup, `m`: 1.2
- Fraction Sold, `s`: The optimization variable, `s ∈ [0, 1]`

2. Formulate Final Wealth (`W`) as a Function of `s`

- **On a Win (`W_win`):** `W_win(s) = (15000 - 1000) + (1-s)*5000 + (s * 1000 * 1.2) = 19000 - 3800s`
- **On a Loss (`W_lose`):** `W_lose(s) = (15000 - 1000) + (s * 1000 * 1.2) = 14000 + 1200s`

3. Construct the Objective Function `f(s)` The objective is to maximize the expected logarithm of wealth: `f(s) = 0.26 * ln(19000 - 3800s) + 0.74 * ln(14000 + 1200s)`

4. Find the Maximum Solving `f'(s) = 0` (or applying the generalized formula from Section 2.2) yields the optimum: `s = 2/3 ≈ 0.6667`

5. Verify the Result The table below compares the Certainty-Equivalent Bankroll (`W_ce`) for various sale percentages, illustrating how the value peaks at the calculated optimum.

Sale Percentage (s)	Certainty-Equivalent Bankroll (W_ce)
0.0% (Play 100%)	$15,155.45
50.0%	$15,214.28
60.0%	$15,215.93
66.7% (Optimal)	**$15,216.03**
70.0%	$15,215.70
100.0% (Sell 100%)	$15,200.00